

Reanalysis, Compatibility, and Correlation in Analysis of Modified Antenna Structures

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A simple computational procedure is synthesized to process changes in the microwave-antenna pathlength-error measure when there are changes in the antenna structure model. The procedure employs structural modification reanalysis methods combined with new extensions of correlation analysis to provide the revised rms pathlength error. Mainframe finite-element-method processing of the structure model is required only for the initial unmodified structure, and elementary postprocessor computations develop and deal with the effects of the changes. Several illustrative computational examples are included. The procedure adapts readily to processing spectra of changes for parameter studies or sensitivity analyses.

I. Introduction

The capability to readily process changes or sequences of changes in antenna finite-element-method (F.E.M.) structure models is useful for design, parameter studies, or design sensitivity analyses. Otherwise these tasks entail major computational effort via ab initio processing. F.E.M. analysis of antenna structures is time-consuming and demanding of mainframe computer resources. It is necessary to solve simultaneous linear load-displacement equations of orders in the thousands. Processing the structural stiffness matrix and the vectors of external loading cases provides the displacements of the F.E.M. nodes. A change in any structural element property or boundary restraint changes the stiffness matrix and therefore normally requires repetition of the lengthy equation-solving operation. This article discusses short-cut approaches that can readily avoid reformulating and repeating the stiffness matrix equation solution for special cases of changing the initial struc-

ture. Following this it will be shown how to synthesize the antenna performance pathlength error measure with only a trivial amount of additional computation. The procedures used here are postprocessor applications that are independent of and require no coding or algorithm changes in the F.E.M. software used to process the initial model. The only interaction with the F.E.M. software is in the desirability of convenient access to the output results.

The approaches considered here that condense the analysis of modified structures depend upon linearity of the load-displacement formulation. Linearity implies that superposition of displacements is also valid when loadings are superimposed. The final response (displacements, member stresses, and forces) is obtained as the sum of the response of the initial system to the known external loadings and the response to particular unit "indicator" loading vectors [1] that are appropriately

scaled to ensure compatibility in the modified system. The scaling methods are derived from the method of "consistent deformations" [2] or extensions under the topic of "structural modification reanalysis" [3-6].

Although linearity permits superposition of linear response quantities from several loading cases, the antenna surface accuracy is more appropriately expressed in terms of the mean square least squares best-fitting pathlength error, or equivalently, the square root (rms) of this quantity. It will be shown here that it is not necessary to recompute the displacements at all the nodes of the antenna surface F.E.M. model by superposition and then to repeat the least-squares method computations to obtain the best-fitting surface for this new set of displacements. That procedure, which is too lengthy to perform for any reasonably sized model except by a computer of substantial capacity, can be replaced by simple postprocessor hand or desk-type calculator analysis. The simplified calculations use the already available mean square pathlength errors for the several loading cases and the correlation coefficients for these loadings to perform the necessary calculations in a few steps.

II. Analysis for Structure Modifications

A. Method of Consistent Deformations

This method will be used for the situation in which the reflector backup tipping structure is analyzed independently of the supporting alidade or pedestal. The tipping structure F.E.M. model has the reflector supported on the elevation axis bearings. However, as is customary, the boundary restraint that would be in the thrust direction of the bearings is omitted. That is, the support provides no restriction of the reflector motion in the direction of the axis (elevation axis) through the bearings. Consequently it is desirable to correct the reflector analysis for the actual restraint of the alidade in this direction. The consistent deformation condition is that the final reflector and alidade displacements along the axis of the bearings must agree.

The definitions below, in which all terms are derived from separate reflector and mount analyses, are used to solve this problem:

e_R = extension of the reflector from bearing to bearing due to the action of the external loading

e_M = extension of the mount from bearing to bearing due to the reactions from the loading on the reflector plus any other loading applied directly to the mount that is also associated with the same reflector loading case

f_R = extensional compliance of reflector for equal and opposite forces applied at each bearing point along the axis of bearings; that is, this compliance is the extension produced by an indicator loading across the bearings

f_M = compliance of the mount for an equilibrating indicator loading

These defined quantities are shown conceptually in Fig. 1. The bearing points are shown as "A" and "B," R_A and R_B are the corresponding reaction forces, and the indicator loading forces are denoted as " P_S ." Here the displacement quantities are arbitrarily shown as if point A is fixed and point B moves to B' , but actually the quantities required are the differences in displacements (extensions) between the final positions of A and B. All quantities are assumed positive as shown and can be in any set of consistent dimensional units.

From superposition, the final extension of the reflector will be the original extension plus a scale factor R times the extension for the reflector model indicator load. The final extension of the mount will be the original plus the same scale factor times the effect for the mount model indicator load. These final extensions must be equal. That is,

$$e_R + Rf_R = e_M + Rf_M \quad (1)$$

or

$$R = \frac{(e_M - e_R)}{(f_R - f_M)} \quad (2)$$

The application is illustrated in the following examples:

Example 1. The following data are available from a 34-m antenna design:

Tipping-Structure-Only Model	Alidade-Only Model
$e_R = -0.71761$	$e_M = -0.09420$
$f_R = 0.54791$	$f_M = -0.76280$

Thus from Eq. (2)

$$R = \frac{(-0.09420 + 0.71761)}{(0.54791 + 0.76280)} = 0.47563$$

The final extensions are:

$$\text{for the reflector } -0.71761 + (0.47563)(0.54791) = -0.4570$$

$$\text{for the alidade } -0.09420 - (0.47563)(0.76280) = -0.4570$$

Analysis of the composite model of reflector combined with alidade found the final extension to be -0.4628. The difference between this and the number computed above is attributed to roundoff and minor differences in the composite model and the stand-alone individual models.

Example 2. This is an example of satisfying more than one compatibility condition for reflector-mount analysis. Figure 2 shows the schematic of a half-structure subreflector model and its supporting mount in which all details not pertinent to this example have been omitted.

For the subreflector model, the external loading is in the positive z-coordinate direction (vertical), and the primary support in this direction is at node 10, but, similar to Example 1, there is no restraint at the node for motion in the x direction. Node 512 is in line vertically with node 10 and has no external restraints.

The mount attaches to the subreflector at nodes 10 and 512. It contains one bar between these nodes (parallel to the z axis). Node 10 is unrestrained and the restraints on node 512 allow motion in the z direction. The assumptions are (a) node 512 of the mount has no stiffness except in the z direction, and (b) displacements of the subreflector in the z direction do not produce other subreflector forces or reactions. With these assumptions there are two redundancies in the reflector and mount system and these provide the associated compatibility conditions. Similar to Example 1, one condition is that the displacements of subreflector and mount at node 10 in the x direction must be the same. The second condition is that the z-direction extensions between nodes 10 and 512 in both models must be the same.

The indicator loadings that are applied to each model are shown in the figure as P_x and P_z . They are applied in opposite senses for the two models since they are required to be consistent with internal equilibrium in the composite system. The following additional definitions are used:

\mathbf{U}_R = vector containing the x displacement of subreflector node 10 as its first component and the extension between nodes 10 and 512 as the second

\mathbf{U}_M = mount displacement vector with the same components as for the subreflector vector above

\mathbf{R} = vector of scale factors to be found; these apply to the indicator loads

\mathbf{F}_R = subreflector compliance matrix for the indicator loadings; row indices correspond with \mathbf{U}_R and column indices correspond with the indicator loads

\mathbf{F}_M = mount compliance matrix with indices as for \mathbf{F}_R

The compliance matrix components are taken to be positive in the case of increasing the node 10 x-displacement component and lengthening the distance between nodes 10 and 512.

With these definitions the matrix compatibility equation becomes

$$\mathbf{U}_R + \mathbf{F}_R \mathbf{R} = \mathbf{U}_M + \mathbf{F}_M \mathbf{R} \quad (3)$$

Rearranging Eq. (3), the following equation is obtained:

$$(\mathbf{U}_R - \mathbf{U}_M) = (\mathbf{F}_M - \mathbf{F}_R) \mathbf{R} \quad (4)$$

which can be solved for \mathbf{R} .

Specific input data from the subreflector and mount analyses and the computed value of \mathbf{R} are given in Table 1. Using either the left side or the right side of Eq. (3), the updated extensions can be computed from the value of \mathbf{R} just found. The result is compared below with that obtained by a composite model analysis of subreflector and mount:

	Computed here	Composite model
x-displacement	0.07103	0.07118
z-extension	-0.02822	-0.02831

B. Parallel Element Method of Structural Modification Reanalysis

This method [6-8] uses superposition and compatibility to provide great simplicity in processing spectra of changes in properties for elemental members of the F.E.M. model. It is particularly simple in both concept and application when applied to the analysis [8] or design [9, 10] of predominately one-dimensional-type rod members used in antenna structures. The method invokes only elementary postprocessor computations that, except for the requirement that the F.E.M. analysis process additional self-equilibrating indicator loads [1] in the usual way, are independent of the F.E.M. software.

The method can be applied for rod, beam, plate, or other elements of the F.E.M. model, but here the application will be restricted to the rod-type elemental member. The concept is that for each particular "parent" member of the structure to

be changed, there is conceptually a "parallel" member attached to the structure in the same way as the parent. The area of the parallel member, positive for additions and negative for reductions, is the change in area of the parent. An independent indicator loading consisting of a pair of unit indicator loads (directed towards each other) is applied at the terminal nodes of each parent member in the original structure. The following is a summary of the algorithms in the notation of and abstracted from [8].

1. Notation.

U_I = the displacement matrix of the initial unmodified structure; the order is m (degrees of freedom) by k (number of external loading columns)

U_M = the displacement matrix to be found for the modified structure

U_D = the change in displacements, which is equal to the displacements for the forces of the parallel members acting as loads on the initial structure

U_S = the displacement matrix of the initial structure for the indicator loadings; the order is $m \times b$ (number of parent member changes)

R = a matrix of scale factors of magnitudes to be found for the indicator loadings; the order is $b \times k$

e_M = the matrix of final extensions of parent and parallel members in the modified structure; the order is $b \times k$

e_I = the matrix of initial extensions of the parent members; the order is $b \times k$

e_S = the matrix of extensions of the parent members for the indicator loadings; the order is $b \times b$

e_0 = a diagonal matrix of extensions of the parallel members when isolated from the structure and loaded by tensile indicator load pairs; the order is $b \times b$

2. Algorithms. It is evident that the displacement of the modified structure is equal to the displacements for the initial structure plus the changes. That is,

$$U_M = U_I + U_D \quad (5)$$

Since the change in displacements is equal to the displacements caused by the indicator loads multiplied by their scale factors, Eq. (5) is rewritten as

$$U_M = U_I + U_S R \quad (6)$$

Similarly for the extensions,

$$e_M = e_I + e_S R \quad (7)$$

Nevertheless, the extension for the isolated parallel members when subjected to the scaled values of the indicator loadings must be the same for compatibility of parent and parallel member extensions, that is,

$$e_M = e_0 R \quad (8)$$

Combining Eqs. (7) and (8) leads to the following expression that can be solved for R ,

$$(e_0 - e_S)R = e_I \quad (9)$$

Once R has been determined, Eqs. (6) and (7) will provide the displacements and extensions of the modified structure.

Computation of the terms needed to formulate Eq. (9) is particularly simple for rod members of the structure. From Hooke's Law the extension of a rod in terms of its internal stress resultant force P , length L , area A , and Young's Modulus E is

$$e = \frac{PL}{AE} \quad (10)$$

Therefore all that is needed from the finite-element program to compute the extension terms are the output vectors of initial and indicator loading internal forces for the various external and indicator loadings. This allows Eq. (10) to be used as a simpler alternative to computing the extensions directly from the displacements of the terminal nodes. The extension of a typical isolated parallel member for an indicator loading of magnitude M (if not of unit magnitude) is

$$e_0 = \frac{ML}{A_D} \quad (11)$$

in which A_D is the area of the parallel member (equal to the change in area of parent member).

It is simple to show that if Eqs. (10) and (11) are used in Eq. (9), and both sides of Eq. (9) are premultiplied by the inverse of a diagonal matrix containing the AE/L terms, the following equation provides an alternative way to solve for R :

$$(A^* - P_S)R = P_I \quad (12)$$

in which A^* is a diagonal matrix containing the MA/A_D term appropriate to each row and P_S and P_I are the matrices of

internal forces of the parent members for the indicator loadings and for the external loadings.

The internal forces \mathbf{P}_M for the modified structure are computed analogously to Eq. (5) as

$$\mathbf{P}_M = \mathbf{P}_I + \mathbf{P}_D \quad (13)$$

where \mathbf{P}_D is the change in force equal to [10]

$$\mathbf{P}_S = (\mathbf{P}_S + \mathbf{I}_S)\mathbf{R} \quad (14)$$

in which \mathbf{I}_S is a quasi-identity matrix with one unity element in each row corresponding to the rows of \mathbf{R} and is null elsewhere.

3. Comments.

- (1) If there is no parent member in the initial structure at a particular row, in Eq. (12) that row can be replaced by the formulation in terms of the extension as given in Eq. (9). A_D becomes the area of the member added.
- (2) If it is desired to remove a member, A_D should be the negative of the parent member area.
- (3) \mathbf{P}_S is the negative of an identity matrix if the set of parent members is statically determinate. If any row of the matrix is null except for a negative unity on the diagonal, the associated parent member is essential to stability and cannot be removed.
- (4) Examination of the \mathbf{P}_S matrix can provide an indication of the redundancy of the parent members. The stronger the off-diagonal coupling, the more redundant.
- (5) It is simple to process spectra of parent member changes because the formulations of Eq. (9) or Eq. (12) remain almost intact. The only terms that change are those that depend upon changes of A_D . It may be appropriate to substitute an arbitrarily small number for A_D that is several orders of magnitude smaller than the area of the parent if there is to be no modification for a particular parent member in one of these variations.

Example 3. Figure 3 is a sketch of the half-model of a 70-m antenna subreflector. The reflecting surface is modeled by plate elements and is stiffened by additional plates in the radial and circumferential directions. Supplementary truss structure behind and above the plates provides a backup and the means for attachment to the external supporting structure. The support system has been modified for illustrative purposes in this example and the vertical (z-axis direction) support system has been replaced by the three spring supports shown at points A, B, and C of the figure. The springs are

represented by rods in the model and the area parameter of the rods is to be varied. The computations are to be performed according to the procedures just given for the effect on two particular external loadings. The loadings are 1.0-g loads in the z- and in the y-coordinate directions.

The data, solution, and a sample check of computed forces and deflections derived from an F.E.M. analysis of the modified model are shown in Table 2. The small differences between the check results are attributed to round-off error and limited numbers of significant figures in the data transferred from the initial F.E.M. analysis. In [8] may be found a discussion of how the change in the loading due to changes in the weights of the modified members could be accounted for if necessary.

III. Correlation Analysis for RF Pathlength Performance of Modified Structures

A. Pathlength Error Vector Computation

The microwave antenna pathlength error computations that employ the deflections provided by the F.E.M. analysis are summarized here for ready reference. The linear relationship between the components of the pathlength error vector of the reflector with the Cartesian coordinate deflection components at the surface nodes was derived in [11]. This provided both the matrix relating pathlength error to deflection and the formulation of the least-squares procedure used to best-fit the deflected surface to an alternative surface that minimized the mean-square pathlength error. This formulation was extended in [12] which provided a single linear transformation matrix to express directly the relationship between the best-fitting pathlength error vector and the deflections. This transformation implicitly incorporated the least-squares fitting parameters and provided a one-to-one transformation from the triad of deflections at each node to the best-fit pathlength error of that node. This relationship is in the form

$$\rho = \mathbf{GU} \quad (15)$$

in which ρ is the pathlength error vector, \mathbf{G} is an invariant matrix essentially containing functions of the direction cosines of the ideal reflecting surface, and \mathbf{U} contains the three-component deflection vector at each node.

Since the deflection vector for a combined loading can be assembled as the superposition of a linear combination of the displacement vectors for a set of independent loadings, a pivotal consequence is that the pathlength error vector can be assembled from the independent pathlength errors in the identical way. To be specific, if \mathbf{C} is a vector of constants, \mathbf{U}_j a matrix containing vectors of deflections for a set of individual loadings, ρ_j a matrix containing the corresponding best-

fit pathlength error vectors, and \mathbf{U} and ρ are the corresponding composite deflection and best-fitting pathlength errors, then when the deflection can be superimposed as

$$\mathbf{U} = \mathbf{U}_j \mathbf{C} \quad (16)$$

it also follows that

$$\rho = \rho_j \mathbf{C} \quad (17)$$

An additional favorable consequence is that superposition relationships similar to Eq. (17) also apply to other linear performance measures such as the least-squares best-fitting parameters, the boresight pointing errors, and subreflector offsets.

B. Mean Square Pathlength Error via Correlation Analysis

It is customary to consider a weighted mean-square pathlength error where the weights for microwave antennas depend upon an illumination factor and also the local area tributary to each node. However, if the weights are appropriately normalized so that they sum to unity, the weighting factors can be omitted for brevity in the following discussions without loss of generality. Then the mean-square pathlength error SS is the inner product

$$SS = \rho^t \rho \quad (18)$$

and the root-mean-square error is

$$rms = (SS)^{1/2} \quad (19)$$

When the pathlength error vector is found by superposition according to Eq. (17) then elementary matrix algebra will show that the mean square can be expressed as

$$SS = \mathbf{C}^t \mathbf{C} \mathbf{V} \mathbf{C} \mathbf{C}^t \mathbf{V} \mathbf{C} \quad (20)$$

in which \mathbf{CV} is the covariance matrix with elements given by

$$CV(i,j) = \rho_i^t \rho_j \quad (21)$$

The covariance matrix can be computed from the triple product of a diagonal matrix \mathbf{RM} of rms values of the best-fit pathlength errors for the independent loadings and a correlation matrix \mathbf{CR} as

$$\mathbf{CV} = \mathbf{RM} \mathbf{CR} \mathbf{RM} \quad (22)$$

In Eq. (22) the diagonal elements of matrix \mathbf{RM} are $rms_1, rms_2, \dots, rms_n$, where n is the number of loads that are superim-

posed. The coefficient of the i th row and j th column of the correlation matrix is defined as

$$CR(i,j) = \frac{\rho_i^t \rho_j}{(rms_i rms_j)} \quad (23)$$

It can be observed that the correlation matrix is symmetrical and has unity on the diagonal. The correlation matrix can be produced most conveniently as a by-product in the initial F.E.M. analysis [13] or alternatively by an independent post-processor.

In summary, postprocessor pathlength error computations for a linear combination of loadings are accomplished by applying Eqs. (22), (20), and (19), in that order.

Example 4. The rms pathlength error for the external loading will be computed for the antenna of Example 1. The following data are available from the tipping-structure-only analysis:

	RMS	Correlation coefficient
External loading	0.01090	-0.4331
Indicator loading	0.005705	

Therefore,

$$\mathbf{RM} = \begin{bmatrix} 0.01090 & 0.00000 \\ 0.00000 & 0.005705 \end{bmatrix}$$

$$\mathbf{CR} = \begin{bmatrix} 1.0000 & -0.4331 \\ -0.4331 & 1.0000 \end{bmatrix}$$

and

$$\mathbf{C} = \begin{pmatrix} 1.00000 \\ 0.47563 \end{pmatrix} \quad (R \text{ found from Ex. 1})$$

Then, from Eq. (22)

$$\mathbf{CV} = \begin{bmatrix} 0.0001188 & -0.0000269 \\ -0.0000269 & 0.0000325 \end{bmatrix}$$

from Eq. (20)

$$SS = 1.0056E-04$$

from Eq. (19)

$rms = 0.0100$ (c.f. 0.0099 from composite model analysis of reflector and mount)

Example 5. This is a supplement to Example 2. The pathlength error for the external loading as modified by the interaction with the mount will be computed here. The data and solution are shown in Table 3. The rms value of 0.0066 shown as the solution in the table was also obtained from the F.E.M. analysis of the composite reflector and mount. Actually there is agreement to within unity in the next (not shown) decimal figure.

If correlation analysis had not been used here, the conventional approach to predicting the rms number would be to take the root sum square (rss) of each of the three independent values times the applicable constant (C vector). That result would have led to the value of 0.0123. This inaccuracy is because the rss method is based upon an identity correlation matrix, which is far from the case here.

C. Pathlength Error Syntheses for Multiple Modified External Loadings

Up to this point the computation of pathlength error via correlation analysis for a modified structure treated the effect of modification for only one external loading condition. Frequently, however, the external loading of interest is a combination of two [12] (in the case of gravity loading on an antenna) or more (with the addition of other environmental cases) loadings. Consequently the previous formulation will be extended to include the cases of multiple external loadings for a modified antenna structure.

From superposition, similar to Eq. (17), the pathlength error vector for the external loading on the modified structure can be expressed as

$$\rho_M = \rho_{GM} C \quad (24)$$

in which ρ_M is the pathlength error vector for the modified structure when subjected to the combined external loading, ρ_{GM} is a matrix of the pathlength error vectors for the individual external loading cases of the modified structure, and C is a vector of combining coefficient factors for the external loading vectors.

Premultiplication of Eq. (24) by its transpose provides the desired mean-square pathlength error SS_M for the modified structure. That is

$$SS_M = C^t C V_M C \quad (25)$$

in which CV_M is the covariance matrix for the modified structure given by

$$CV_M = \rho_{GM}^t \rho_{GM} \quad (26)$$

It can be observed that once the modified covariance matrix CV_M is obtained it is trivial to complete the solution by means of Eq. (25). Consequently, the remainder of this discussion will concentrate on deriving an expression for this matrix.

External loading pathlength vectors that have been modified by the parallel element method can be expressed in terms of a matrix of unmodified pathlength error vectors ρ_G , a matrix ρ_I of pathlength errors for the indicator loadings, and R, the matrix of scale factors for the indicator loadings. Therefore, similarly to Eqs. (6) and (8), and with superposition in the form of Eq. (17), the modified pathlength error is

$$\rho_{GM} = \rho_G + \rho_I R \quad (27)$$

In the equation above, all the pathlength vectors are the least-squares best-fitting vectors.

Using Eq. (27) in Eq. (26) it can be shown, with some multiplication and rearrangement, that the desired covariance matrix can be expressed as the sum of four matrices. To do this, the following matrices that are all determined from F.E.M. analysis of the unmodified structure are defined:

RM_G = the diagonal matrix of rms values for the external loading; CR_G is the associated matrix of correlation coefficients

RM_I = the diagonal matrix of rms values for the indicator loads; CR_I is the associated matrix of correlation coefficients

CR_{GI} = the matrix of correlation coefficients for the external loadings with respect to the indicator loading

The above definitions are used in the computation of the following covariance matrices

$$CV_G = RM_G CR_G RM_G \quad (28)$$

$$CV_I = RM_I CR_I RM_I \quad (29)$$

$$CV_{GI} = RM_G CR_{GI} RM_I \quad (30)$$

Finally, omitting the manipulations, the following expression as the sum of four matrices can be developed for CV_M :

$$CV_M = CV_G + CV_{GI} R + (CV_{GI} R)^t + R^t CV_I R \quad (31)$$

The modified covariance matrix is square and of order equal to the number of external loads. The diagonal elements are the mean-square pathlength errors for the external loads on the modified structure and the off-diagonals are the pairwise covariances for these loads.

Example 6. The covariance matrix of the modified structure of Example 3 will be computed in this final example. The data and solution are shown in Table 4. The modified covariance matrix CV_M shown as the solution in Table 4 can be used as described in the preceding paragraph to furnish the rms value for the external loadings and their correlation coefficient. These are compared below with those obtained by a full F.E.M. analysis of the modified structure.

	Computed here	F.E.M. analysis
External z-loading rms	0.03922	0.03917
External y-loading rms	0.00862	0.00861
Correlation coefficient	0.8755	0.8753

IV. Summary

The synthesis of a readily applied procedure to compute the performance parameters of modified antenna structures has been presented. All the necessary computations can conveniently be developed by desk calculator or personal computer postprocessing. The input data needed consist of conventional mainframe computer analysis output for the unmodified structure. The synthesized procedure uses short-cut structure modification reanalysis methods to avoid reprocessing the modified finite-element-method structure model. Then, changes in the antenna root-mean-square pathlength error performance measure are computed for the modified structure by extended methods of correlation analysis. The complete modification and correlation analysis synthesis readily accommodates the processing of spectra of changes in the antenna structure for purposes such as for parameter studies or for design sensitivity analyses.

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Table 1. Data and solution for Example 2

Data	Solution
$U_R = \begin{pmatrix} 0.87642 \\ -0.32976 \end{pmatrix}$	
$U_M = \begin{pmatrix} 0.05865 \\ 0.0 \end{pmatrix}$	$F_M - F_R = \begin{bmatrix} -0.40864 & 0.129000 \\ 0.129000 & -0.060766 \end{bmatrix}$
$F_R = \begin{bmatrix} 0.394478 & -0.129000 \\ -0.129000 & 0.052867 \end{bmatrix}$	$U_M - U_R = \begin{pmatrix} 0.81777 \\ -0.32976 \end{pmatrix}$
$F_M = \begin{bmatrix} -0.014172 & 0.0 \\ 0.0 & -0.007899 \end{bmatrix}$	$R = \begin{pmatrix} -0.873343 \\ 3.57202 \end{pmatrix}$

Table 2. Data and solution for Example 3

Data:

Spring	External loading, \mathbf{P}_I		Self-equilibrating indicator loading, \mathbf{P}_S ($M = 1000$)		
	Z, 1.0-g	Y, 1.0-g	Load at A	Load at B	Load at C
A	-2451.0	8.6	-474.5	-262.8	-262.8
B	-2318.0	-625.3	-262.8	-868.6	131.4
C	-2031.0	616.8	-262.8	131.4	-868.6

Spring	Relative property change, A_D/A
A	-0.98
B	-0.99
C	0.20

Solution:

Spring	\mathbf{A}^* (diagonal elements only, Eq. 10)
A	-0.10220408M = -102.20408
B	-0.10101010M = -101.01010
C	0.50000000M = 5000.00000
$\mathbf{A}^* - \mathbf{P}_S$	$\begin{bmatrix} -545.908 & 262.800 & 262.800 \\ 262.800 & -141.501 & -131.400 \\ 262.800 & -131.400 & 5868.600 \end{bmatrix}$
\mathbf{P}_I	$\begin{bmatrix} -2451.0 & 8.6 \\ -2318.0 & -625.3 \\ -2031.0 & 616.8 \end{bmatrix}$
\mathbf{R}	$\begin{bmatrix} 116.718 & 19.979 \\ 233.475 & 41.397 \\ -0.345 & 0.137 \end{bmatrix}$ (See Eq. 12)

Checks:

Internal force check					
Spring	Computed here (Eq. 14) External loading		Finite-element analysis External Loading		
	Z, 1.0-g	Y, 1.0-g	Z, 1.0-g	Y, 1.0-g	
A	-2382	-407.7	-2378	-407.1	
B	-2358	-418.2	-2355	-417.5	
C	-2071	823.9	-2068	824.6	

Deflection check					
Node index	Computed here (Eq. y) External loading		Finite-element analysis External loading		
	Z, 1.0-g	Y, 1.0-g	Z, 1.0-g	Y, 1.0-g	
124-2	-0.11722	-0.01454	-0.11706	-0.01451	
124-3	3.12309	0.54772	3.1185	0.54688	

Table 3. Data and solution for Example 5

<i>Data:</i>			
Loading case			
	rms	Correlation coefficients	
		Indicator X	Indicator Z
External	0.006647	0.9725	-0.8632
Indicator X	0.002335		-0.8297
Indicator Z	0.002845		

Loading factors			
$C = \begin{pmatrix} 1.000000 \\ -0.873342 \\ 3.57202 \end{pmatrix}$	$(R(1))$		

<i>Solution:</i>			
$RM = \begin{bmatrix} 0.0066471 & 0.0 & 0.0 \\ 0.0 & 0.002335 & 0.0 \\ 0.0 & 0.0 & 0.002845 \end{bmatrix}$			
$CR = \begin{bmatrix} 1.0 & 0.9725 & -0.8632 \\ 0.9725 & 1.0 & -0.8297 \\ -0.8632 & -0.8297 & 1.0 \end{bmatrix}$			
$CV = 10^{-4} \times \begin{bmatrix} 0.441839 & 0.15094 & -0.163240 \\ 0.15094 & 0.05452 & -0.055118 \\ -0.163240 & -0.055118 & 0.080940 \end{bmatrix}$			

rms = 0.0066

Table 4. Data and solution for Example 6

Data:

Loading case		rms
External Z		0.3359E-02
External Y		0.4049E-02
Indicator A		0.3590E-03
Indicator B		0.2391E-03
Indicator C		0.2499E-03

Loading	Correlation			
	Ext. Y	Ind. A	Ind. B	Ind. C
External Z	-0.1103	0.7464	-0.5104	-0.5839
External Y		-0.0494	0.1740	-0.0955
Indicator A			-0.7201	-0.7499
Indicator B				0.0776

Solution:

$$\begin{aligned}
 \mathbf{RM}_G &= \begin{bmatrix} 0.3359E-02 & 0.0 \\ 0.0 & 0.4049E-02 \end{bmatrix} & \mathbf{R} &= \begin{bmatrix} 116.718 & 19.979 \\ 233.475 & 41.397 \\ -0.345 & 0.137 \end{bmatrix} & \text{(from Table 2)} \\
 \mathbf{RM}_I &= \begin{bmatrix} 0.3590E-03 & 0.0 & 0.0 \\ 0.0 & 0.2391E-03 & 0.0 \\ 0.0 & 0.0 & 0.2499E-03 \end{bmatrix} & \mathbf{CR}_G &= \begin{bmatrix} 1.0000 & -0.1103 \\ -0.1103 & 1.0000 \end{bmatrix} \\
 \mathbf{CV}_G &= 10^{-4} \times \begin{bmatrix} 0.112829 & -0.015500 \\ -0.015002 & 0.163944 \end{bmatrix} & \mathbf{CR}_I &= \begin{bmatrix} 1.0000 & -0.7201 & -0.7477 \\ -0.7201 & 1.0000 & 0.0776 \\ -0.7477 & 0.0776 & 1.0000 \end{bmatrix} \\
 \mathbf{CV}_{GI} &= 10^{-6} \times \begin{bmatrix} 0.900070 & -0.409921 & -0.490134 \\ -0.071807 & 0.168452 & -0.096631 \end{bmatrix} & \mathbf{CR}_{GI} &= \begin{bmatrix} 0.7464 & -0.5104 & -0.5839 \\ -0.0494 & 0.1740 & -0.0955 \end{bmatrix} \\
 \mathbf{CV}_M &= 10^{-2} \times \begin{bmatrix} 0.153825 & 0.029600 \\ 0.029600 & 0.007430 \end{bmatrix} & \mathbf{CV}_I &= 10^{-6} \times \begin{bmatrix} 0.128880 & -0.061812 & -0.067072 \\ -0.061811 & 0.057169 & 0.004637 \\ -0.067079 & 0.004637 & 0.062450 \end{bmatrix}
 \end{aligned}$$

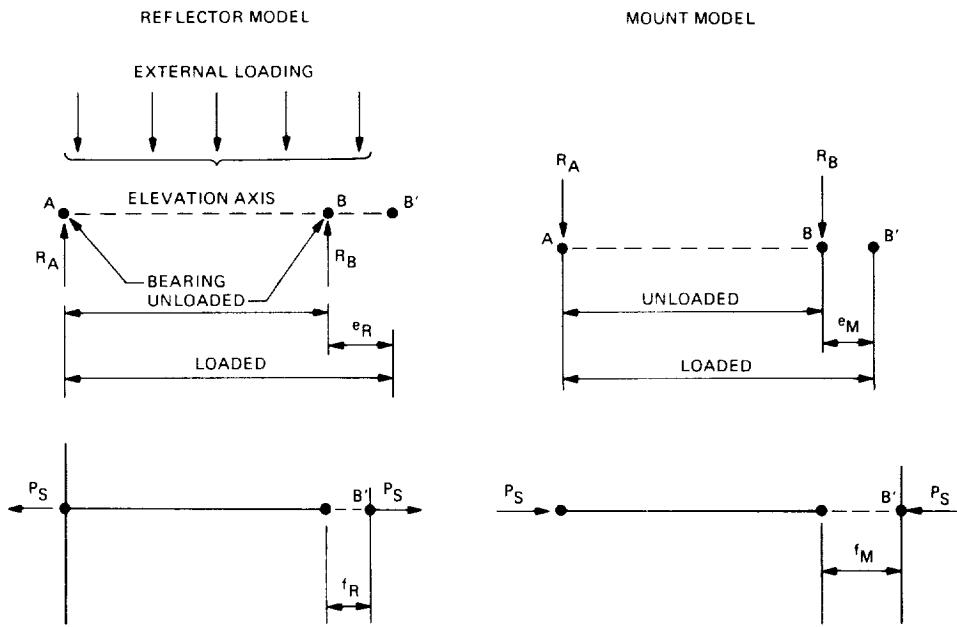


Fig. 1. Schematic for reflector-mount axis compatibility, Example 1.

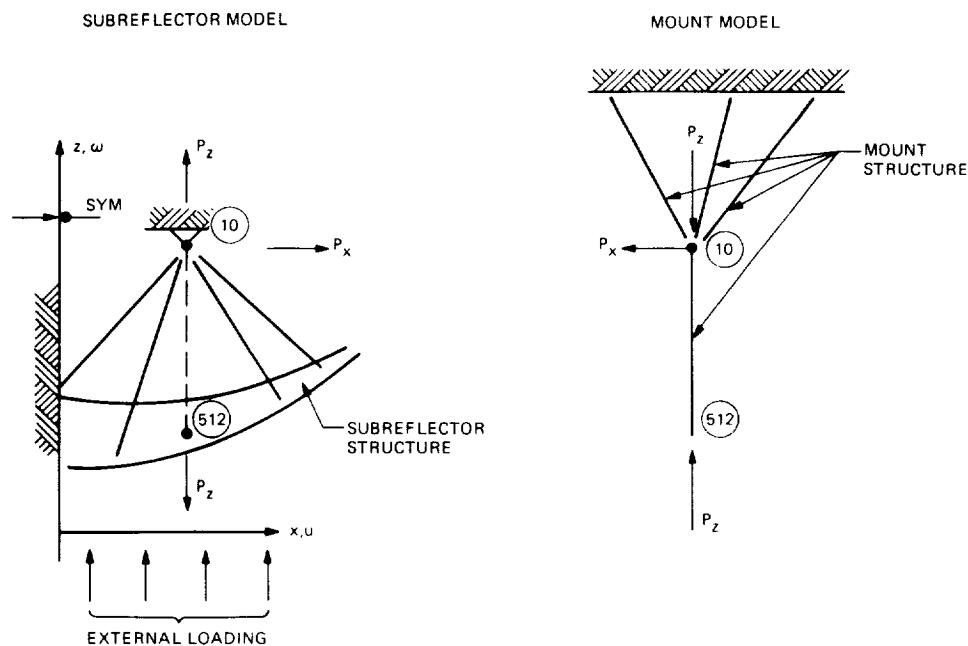


Fig. 2. Schematic for subreflector-mount model, Example 2.

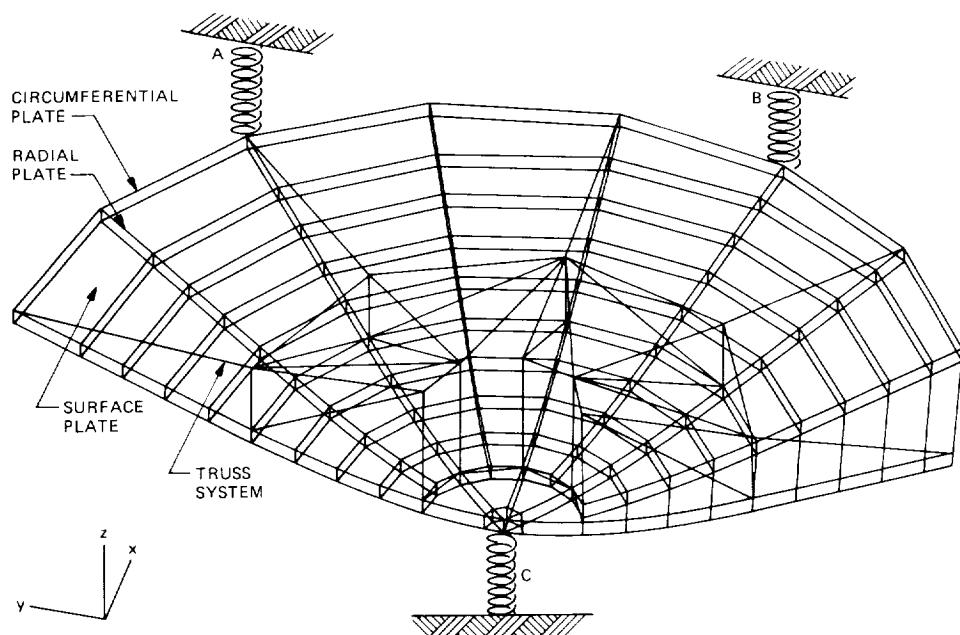


Fig. 3. Subreflector half-structure model, Example 3.

